PACKING AND HAUSDORFF MEASURES OF CANTOR SETS ASSOCIATED WITH SERIES

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(JOINT WORK WITH KATHRYN HARE AND FRANKLIN MENDIVIL)

ABSTRACT

In 1989 Morán introduced -for a given summable sequence $a = (a_n)$- the associated Cantor sets to a, $C_a = \{\sum_{i=1}^{\infty} a_i \varepsilon_i : \varepsilon_i = 0, 1\}$. Assuming a suitable separation condition, in [Mo 94] Morán related the h-Hausdorff measure of C_a to the quantities $R_n = \sum_{i>n} \|a_i\|$.

In this paper, we generalize Morán's sum set notion to permit a greater diversity in the geometry (for instance, unlike Morán's sets, our generalized sum sets can have Hausdorff dimension greater than one).

We obtain the analogue of Morán's results on h-Hausdorff measures for these generalized sum sets and prove dual results for h-packing measures.

We show that for any of these sum sets there is a doubling dimension function h for which the sum set has both finite and positive h-Hausdorff and h-packing measure.

We prove that the class is big enough in the sense that for a given α less than the Hausdorff dimension (or β less than the packing dimension) there is a sum subset that has Hausdorff dimension α (or packing dimension β). In fact, there is even a sum subset with both Hausdorff dimension α and packing dimension β provided α/β is dominated by the ratio of the Hausdorff dimension to the packing dimension of the original set. Furthermore, if the Hausdorff and/or packing measure is finite and positive (in the corresponding dimension), then we can choose this sum subset to have finite and positive Hausdorff and/or packing measure.Moreover, we examine the validity of the same results when the dimension is given for a function h instead of a real number α.

References

[Mo 89] M. Morán, "Fractal series", *Mathematika* **36** (1989), 334-348.

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