PACKING AND HAUSDORFF MEASURES OF CANTOR SETS ASSOCIATED WITH SERIES

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(JOINT WORK WITH KATHRYN HARE AND FRANKLIN MENDIVIL)

ABSTRACT

In 1989 Morán introduced -for a given summable sequence $a = (a_n)$- the associated Cantor sets to $a$, $C_a = \{ \sum_{i=1}^{\infty} a_i \varepsilon_i : \varepsilon_i = 0, 1 \}$. Assuming a suitable separation condition, in [Mo 94] Morán related the $h$-Hausdorff measure of $C_a$ to the quantities $R_n = \sum_{i>n} \|a_i\|$.

In this paper, we generalize Morán’s sum set notion to permit a greater diversity in the geometry (for instance, unlike Morán’s sets, our generalized sum sets can have Hausdorff dimension greater than one).

We obtain the analogue of Morán’s results on $h$-Hausdorff measures for these generalized sum sets and prove dual results for $h$-packing measures.

We show that for any of these sum sets there is a doubling dimension function $h$ for which the sum set has both finite and positive $h$-Hausdorff and $h$-packing measure.

We prove that the class is big enough in the sense that for a given $\alpha$ less than the Hausdorff dimension (or $\beta$ less than the packing dimension) there is a sum subset that has Hausdorff dimension $\alpha$ (or packing dimension $\beta$). In fact, there is even a sum subset with both Hausdorff dimension $\alpha$ and packing dimension $\beta$ provided $\alpha/\beta$ is dominated by the ratio of the Hausdorff dimension to the packing dimension of the original set. Furthermore, if the Hausdorff and/or packing measure is finite and positive (in the corresponding dimension), then we can choose this sum subset to have finite and positive Hausdorff and/or packing measure.Moreover, we examine the validity of the same results when the dimension is given for a function $h$ instead of a real number $\alpha$.

References


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